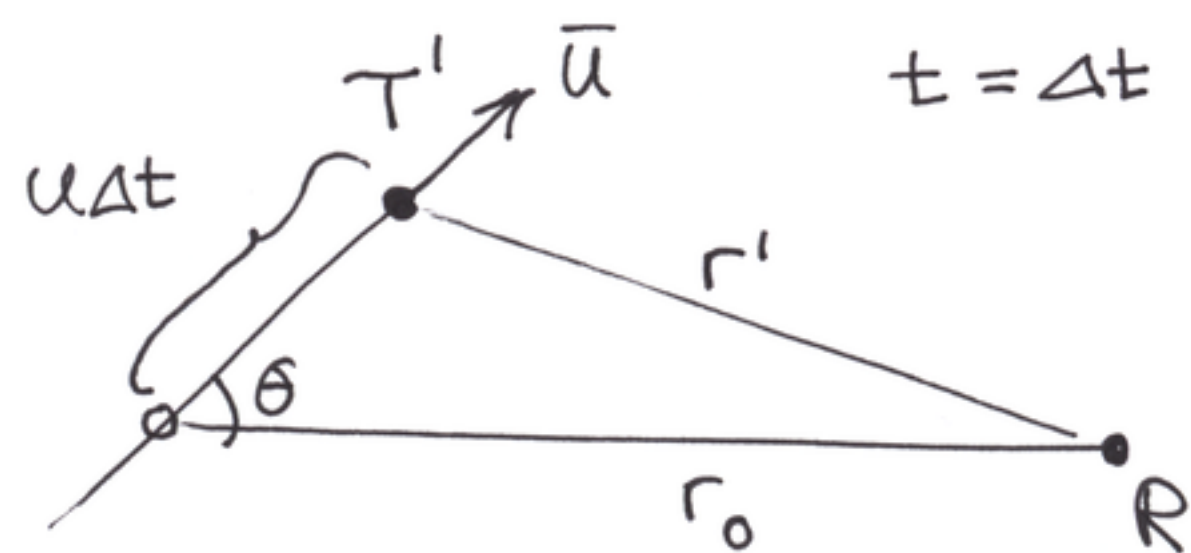
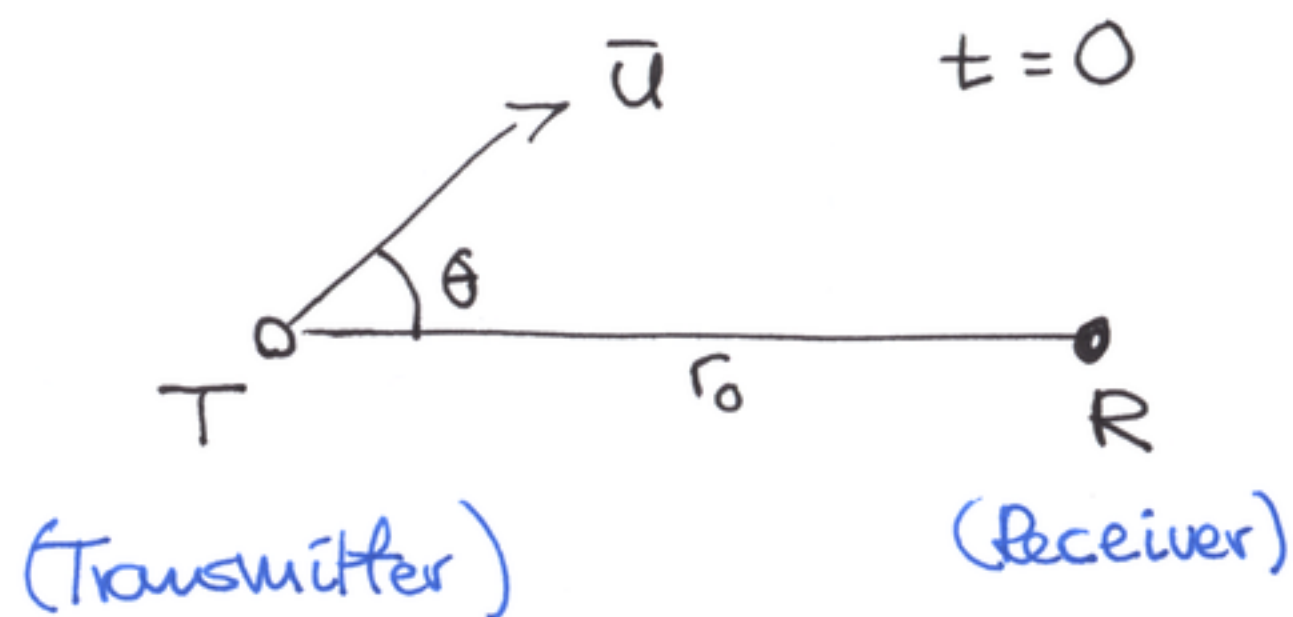


Dopplerhrif



Bylgja frá T klukkan $t=0$
kemur til R kl. $t_1 = \frac{r_0}{c}$

bylgja kl. $t = \Delta t$ frá T' kemur
til R kl.

$$t_2 = \Delta t + \frac{r'}{c}$$

$$= \Delta t + \frac{1}{c} \sqrt{r_0^2 + (u\Delta t)^2 - 2r_0(u\Delta t)\cos\theta}$$

$$\approx \Delta t + \frac{r_0}{c} \left(1 - \frac{u\Delta t}{r_0} \cos\theta \right)$$

ef $r_0 \gg u\Delta t$

Tíðamunur merkanna í R

er

$$\Delta t' = t_2 - t_1 \approx \Delta t + \frac{r_0}{c} \left(1 - \frac{u\Delta t}{r_0} \cos\theta \right) - \frac{r_0}{c}$$

$$= \Delta t \left(1 - \frac{u}{c} \cos\theta \right)$$

því er $\Delta t'$ mælt við hljóðnemann ekki sama og Δt mælt við uppsprettuna

Hljóðneminn heyrir tíðuna

$$f' = \frac{1}{\Delta t'} \approx \frac{1}{\Delta t \left(1 - \frac{u}{c} \cos \theta\right)}$$

$$= \frac{f}{\left(1 - \frac{u}{c} \cos \theta\right)} \approx f \left(1 + \frac{u}{c} \cos \theta\right)$$

$$\text{Ef } \left(\frac{u}{c}\right)^2 \ll 1$$

Raudvík, Blávík

Þverrafsegulbylgjur

Rafsegulbylgja í z-átt
var með fasor

$$\bar{E}(z) = \bar{E}_0 e^{-ikz}$$

fyrir almenna stefnu fæm við

$$E(\bar{x}) = \bar{E}_0 e^{-ik_x x - ik_y y - ik_z z}$$

ef

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon$$

eins og jafna Helmholtz
kretst.

Stílgreinum bylgjuvígur

$$\bar{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z = k \hat{a}_n$$

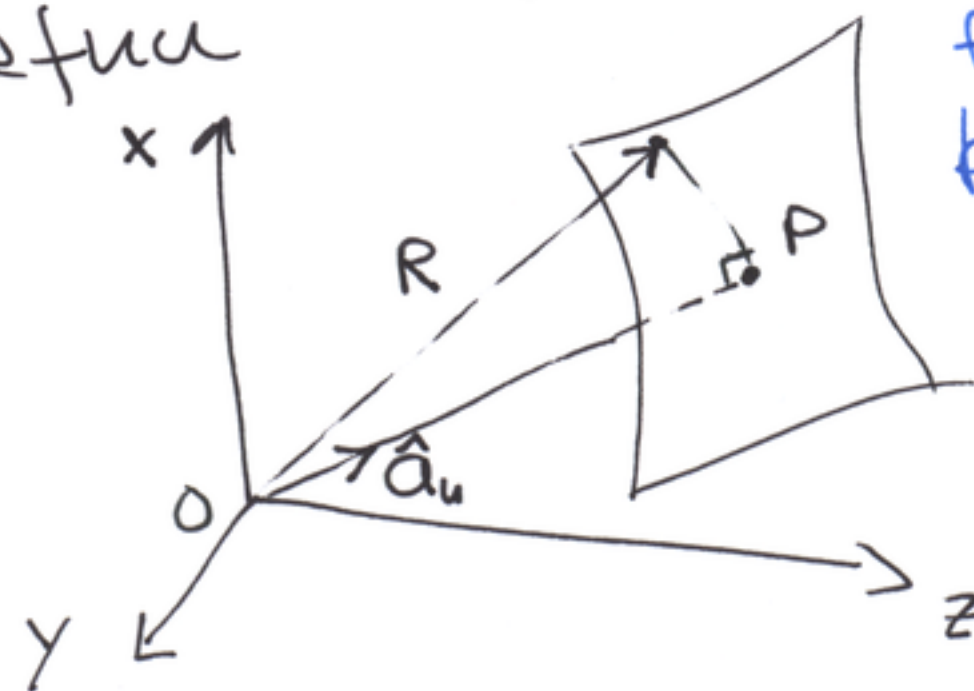
og geisla vígur

$$\bar{R} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$$

þá fæst

$$\bar{E}(\bar{R}) = \bar{E}_0 e^{-i\bar{k} \cdot \bar{R}} = \bar{E}_0 e^{-ik \hat{a}_n \cdot \bar{R}}$$

$\hat{a}_n$ er einingervígur í útbreiddis-
stefnu



flötur bylgju
þvert á $\hat{a}_n$

$$k_x = \vec{k} \cdot \hat{a}_x = k \hat{a}_n \cdot \hat{a}_x$$

og samskonar fyrir $y, z, \dots$

stefnu kósínus fyrir $\hat{a}_n$

$$\hat{a}_n \cdot \vec{R} = \text{fasti} = |\vec{OP}|$$

er jafna sléttumör
þvert á $\hat{a}_n$, með
fastan fasa og útslag

Engin hleðsla á útbreðslusvði

$$\rightarrow \vec{\nabla} \cdot \vec{E} = 0$$

$$\rightarrow \vec{E}_0 \cdot \vec{\nabla} (e^{-ik\hat{a}_n \cdot \vec{R}}) = 0$$

$$\vec{\nabla} (e^{-ik\hat{a}_n \cdot \vec{R}}) = \left(\hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + \hat{a}_z \frac{\partial}{\partial z} \right) e^{-i(k_x x + k_y y + k_z z)}$$

$$= -i(\hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z) e^{-i(k_x x + \dots + k_z z)}$$

$$= -ik\hat{a}_n e^{-ik\hat{a}_n \cdot \vec{R}}$$

E_n

$$\rightarrow -ik(\vec{E}_0 \cdot \hat{a}_n) e^{-ik\hat{a}_n \cdot \vec{R}} = 0$$

sem verður aðeins með

$$\hat{a}_n \cdot \vec{E}_0 = 0$$

$\vec{E}_0$ er þvert á útbreðslu-
stefnu $\hat{a}_n$

Sögulsviðrið finnum við með

$$\vec{H} = \frac{1}{-i\omega\mu} \vec{\nabla} \times \vec{E}$$

$$\rightarrow \vec{H}(\vec{R}) = \frac{1}{\eta} \hat{a}_n \times \vec{E}(\vec{R})$$

með

$$\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}}$$

Þetta

$$\vec{H}(\vec{R}) = \frac{1}{\eta} (\hat{a}_n \times \vec{E}_0) e^{-ik\hat{a}_n \cdot \vec{R}}$$

5
Svo eins og þúast mátti við á svæði án ρ og J eru $\vec{E}$ og $\vec{H}$ hornrétt og líka á útbreiddis stefnum $\hat{a}_n$

Skautun

skoðum

$$\vec{E}(z) = \hat{a}_x E_1(z) + \hat{a}_y E_2(z)$$

$$= \hat{a}_x E_{10} e^{-ikz} - \hat{a}_y i E_{20} e^{-ikz}$$

Sett saman er tveimur línulega skautunum þáttum, annar er 90° á eftir hinum í fasa

Skóðum ~~þessa~~ bylgju í föstum
tímamarkpunkti

$$\bar{E}(z, t) = \text{Re} \left\{ [\hat{a}_x E_1(z) + \hat{a}_y E_2(z)] e^{i\omega t} \right\}$$

$$= \hat{a}_x E_{10} \cos(\omega t - kz) + \hat{a}_y E_{20} \cos(\omega t - kz - \pi/2)$$

Skóðum stefnuþvefningu $\bar{E}(z, t)$ þ. $z=0$
en tímamarkliður

$$\begin{aligned} \bar{E}(0, t) &= \hat{a}_x E_1(0, t) + \hat{a}_y E_2(0, t) \\ &= \hat{a}_x E_{10} \cos \omega t + \hat{a}_y E_{20} \sin \omega t \end{aligned}$$

$$\sqrt{1 - \left(\frac{E_1(0, t)}{E_{10}} \right)^2}$$

$$\rightarrow \cos \omega t = \frac{E_1(0, t)}{E_{10}}$$

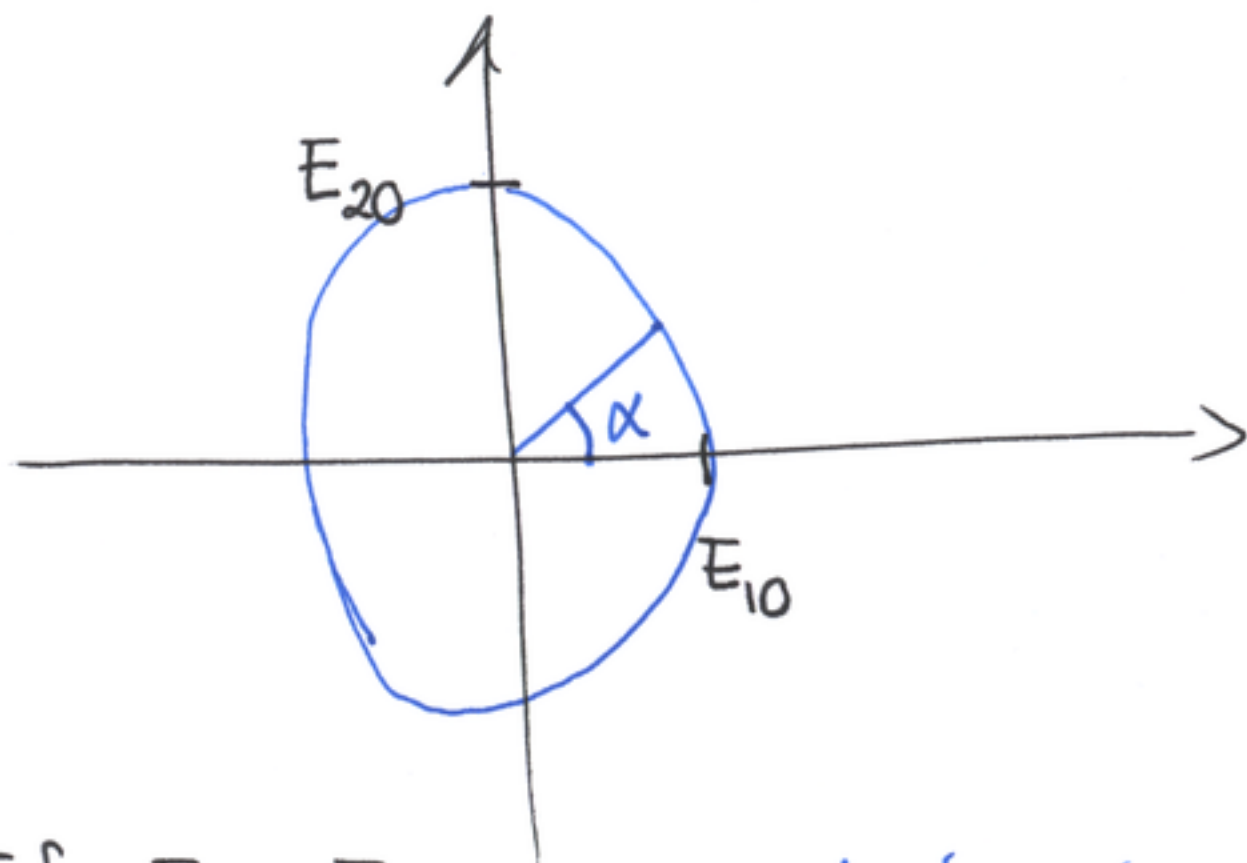
$$\sin \omega t = \frac{E_2(0, t)}{E_{20}} = \sqrt{1 - \cos^2 \omega t}$$

p.a.

$$\left\{ \frac{E_2(0,t)}{E_{20}} \right\}^2 + \left\{ \frac{E_1(0,t)}{E_{10}} \right\}^2 = 1$$

jafna sporbaugs (ellipsu)

ellipsuskantun



Ef $E_{20} = E_{10}$

hringskantun

$$\alpha = \arctan\left(\frac{E_2(0,t)}{E_1(0,t)}\right) =$$

$$\arctan(\tan \omega t) = \omega t$$

Hogri handar hrings ~~sea~~
ellipsu skantun

(jökvað hringskantun)

Vinstri handar hringskantun
föst með

$-\omega t$

snúningi

linuleg skantun $\bar{E}(z) = \hat{a}_x E_0 e^{-ikz}$

$$\bar{E}(z) = \bar{E}_{rc}(z) + \bar{E}_{lc}(z)$$

með

$$\bar{E}_{rc}(z) = \frac{E_0}{2} (\hat{a}_x - i\hat{a}_y) e^{-ikz}$$

$$\bar{E}_{lc}(z) = \frac{E_0}{2} (\hat{a}_x + i\hat{a}_y) e^{-ikz}$$

Flatarbylgjur í efni
með ortu tapi

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$$\nabla^2 \bar{E} + k_c^2 \bar{E} = 0$$

$$k_c = \omega \sqrt{\mu \epsilon_c} \in \mathbb{C}$$

Verja að skilgreina

$$\gamma = i k_c = i \omega \sqrt{\mu \epsilon}$$

og ef

$$\epsilon_c = \epsilon - i \frac{\gamma}{\omega}$$

$$\rightarrow \gamma = \alpha + i\beta = i \omega \sqrt{\mu \epsilon} \left(1 + \frac{\gamma}{i \epsilon \omega} \right)^{1/2}$$

$$\text{eða með } \epsilon_c = \epsilon' - i \epsilon''$$

fóð

$$\gamma = \alpha + i\beta = i \omega \sqrt{\mu \epsilon'} \left(1 - i \frac{\epsilon''}{\epsilon'} \right)^{1/2}$$

α og β eru raun og þverhlutar

γ

án taps er

$$\gamma = 0, \epsilon'' = 0, \epsilon = \epsilon'$$

$$\alpha = 0, \beta = k = \omega \sqrt{\mu \epsilon'}$$

Jafna Helmholtz er hér

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0$$

og fyrir slætta bylgju í
Z-stefnu línulega skautuð
í X-átt

$$\begin{aligned}\bar{E} &= \hat{a}_x E_x = \hat{a}_x E_0 e^{-\gamma z} \\ &= \hat{a}_x E_0 e^{-\alpha z} e^{-i\beta z}\end{aligned}$$

α, β eru báðar jákvæðar
stærðir (kemur í lýs)

α : dofnumar fasti

β : fasa fasti

Rafsvanir með litlu tapi

$$\epsilon' \gg \epsilon'' \quad \text{þá} \quad \frac{\gamma}{\omega \epsilon} \ll 1$$

$$\gamma = \alpha + i\beta \approx i\omega\sqrt{\mu\epsilon'} \left(1 - \frac{i\epsilon''}{2\epsilon'} + \frac{1}{8} \left(\frac{\epsilon''}{\epsilon'} \right)^2 \right)$$

$$\rightarrow \alpha \approx \frac{\omega \epsilon''}{2} \sqrt{\frac{\mu}{\epsilon'}} \quad \leftarrow \text{línulegt m. } \omega$$

$$\beta \approx \underbrace{\omega\sqrt{\mu\epsilon'}}_{\text{hafnum}} \left[1 + \frac{1}{8} \left(\frac{\epsilon''}{\epsilon'} \right)^2 \right]$$

$$\eta = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon'}} \left(1 - i \frac{\epsilon''}{\epsilon'}\right)^{-1/2} \approx \sqrt{\frac{\mu}{\epsilon'}} \left(1 + i \frac{\epsilon''}{2\epsilon'}\right)$$

↑
Hlutfall E_x og H_y hér →

rafsvæðid og segulsvæðid
eru ekki í fasa eins og
í efni án taps

fasa hraðinn er nánna

$$v_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{\mu\epsilon'}} \left[1 - \frac{1}{8} \left(\frac{\epsilon''}{\epsilon'}\right)^2\right]$$

← minnkæð fasa hraði
vegna taps

α - þáttur k_c berst
ekki í efni

Gæður leiðari

$$\frac{\tau}{\omega \epsilon} \gg 1$$

$$\gamma = i\omega \sqrt{\mu \epsilon} \left(1 + \frac{\tau}{i\omega \epsilon}\right)^{1/2}$$

$$\approx i\omega \sqrt{\mu \epsilon} \sqrt{\frac{\tau}{i\omega \epsilon}}$$

$$= \sqrt{i} \sqrt{\omega \mu \tau} = \frac{1+i}{\sqrt{2}} \sqrt{\omega \mu \tau}$$

$$\rightarrow \gamma = \alpha + i\beta \approx (1+i) \sqrt{\pi f \mu \tau}$$

$$\rightarrow \alpha = \beta = \sqrt{\pi f \mu \tau}$$

(11)

$$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} \approx \sqrt{\frac{i\omega \mu}{\tau}}$$

$$= (1+i) \sqrt{\frac{\pi f \mu}{\tau}} = (1+i) \frac{\alpha}{\tau}$$

→ Segulsviðid er á eftir
rafsviðinu í fasa
um $\frac{\pi}{4}$

fasafræðin

$$u_p = \frac{\omega}{\beta} \approx \sqrt{\frac{2\omega}{\mu \tau}}$$

í réttu hlutfelli við
 $\sqrt{f}$ og $\frac{1}{\sqrt{\tau}}$

fyrir góðan leiðara eins
og kapa fast að

$$u_p \approx 720 \text{ m/s}$$

fyrir $f = 3 \text{ MHz}$

Dofnumér er líka stert
 $\alpha = \beta$, bylgjan dofnað
niður í $e^{-1} \rightarrow$

skilgreinir lengd

$$S = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$= \frac{1}{\beta} = \frac{\lambda}{2\pi}$$

Skündýpt (ekki skun....)

$\rightarrow$ fyrir örbylgjur er
skündýptin orðin lítil

Sjá töflu 8-1

Gull

$$S \approx 0,0025 \text{ mm}$$

fyrir $f = 1 \text{ GHz}$