

TE-bylgjur milli samstíða plötua

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$$E_z = 0, \quad x\text{-samhverfa (færslu)}$$

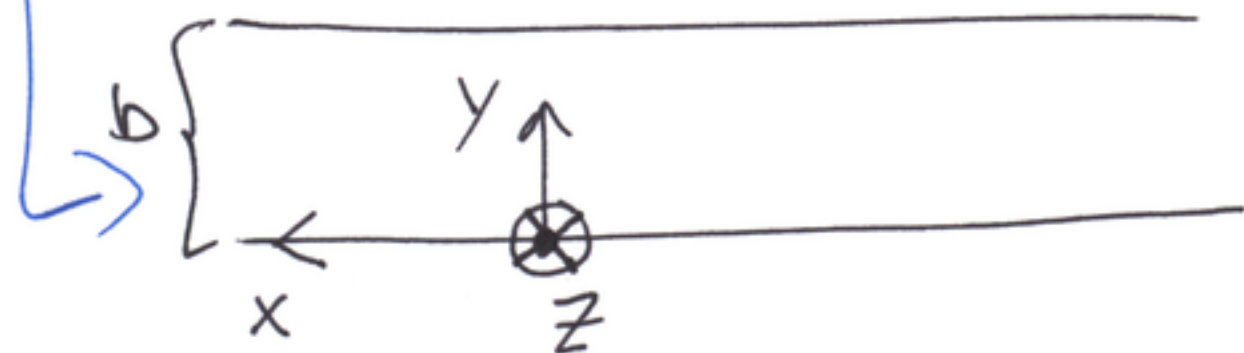
$$\hookrightarrow \left(\frac{d^2}{dy^2} + h^2 \right) H_z^0(y) = 0$$

$$H_z(y, z) = H_z^0(y) e^{-\gamma z}$$

$$E_x = 0 \text{ á plötunum}$$

Maxwell gaf

$$E_x^0 = - \frac{i\omega\mu}{h^2} \frac{\partial H_z^0}{\partial y}$$



$$\frac{dH_z^0(y)}{dy} = 0 \quad \text{fyrir } y=0, b$$

$$H_z^0(y) = B_n \cos\left(\frac{n\pi y}{b}\right)$$

Adrir þættir eru (Maxwell)

$$H_y^0(y) = \frac{\gamma}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$E_x^0(y) = \frac{i\omega\mu}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

↑ Eins og fyrir TM

Orkuflutnings hraði

Vegna þröskulda í tíðni
er ekki víst að grúpuhraði
sé vel skilgreindur fyrir
bylgjubúðara

→ Orkuflutnings hraði

$$v_{\text{en}} = \frac{(P_z)_{\text{ave}}}{W_{\text{ave}}}$$

með

$$(P_z)_{\text{ave}} = \int_S \overline{S_z} \cdot d\overline{S}$$

Meðal talsaflið í þverstundi
 S

og

$$W_{\text{ave}} = \int_S [(W_e)_{\text{ave}} + (W_m)_{\text{ave}}] dS$$

meðal orku geymd í
lengdar einingu buðara

↑ Einingar eru þá réttar
fyrir v_{en}

Meðal tal m. t. t. tíma

Domi

3

Rechnung von μ für TM_n

$$E_z^0(y) = A_n \sin\left(\frac{n\pi y}{b}\right)$$

$$H_x^0(y) = \frac{i\omega\epsilon}{h} A_n \cos\left(\frac{n\pi y}{b}\right)$$

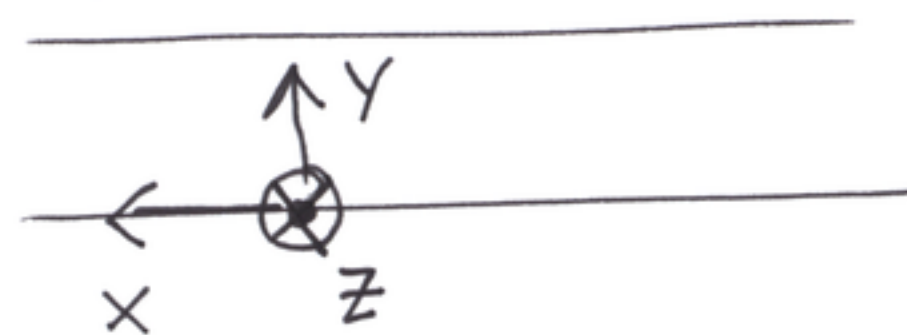
$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos\left(\frac{n\pi y}{b}\right), \quad \gamma = i\beta$$

$$\overline{S}_{ave} = \frac{1}{2} \operatorname{Re}(\overline{E} \times \overline{H}^*)$$

$$= \frac{1}{2} \operatorname{Re}(-\hat{a}_z E_y^0 H_x^{0*} + \hat{a}_y E_z^0 H_x^{0*})$$

$$\rightarrow \overline{S}_{ave} \cdot \hat{a}_z = -\frac{1}{2} \operatorname{Re}(E_y^0 H_x^{0*})$$

$$= \frac{\omega\epsilon\beta}{2h^2} A_n^2 \cos^2\left(\frac{n\pi y}{b}\right)$$



$$(P_z)_{ave} = \int_0^b \overline{S} \cdot \hat{a}_z dy$$

$$= \frac{\omega\epsilon\beta b}{4h^2} A_n^2$$

$\hat{a}$ sinngemäß
wieder (x-Steuer)

$$(W_e)_{ave} = \frac{\epsilon}{4} \text{Re} (\bar{E} \cdot \bar{E}^*)$$

$$(W_m)_{ave} = \frac{\mu}{4} \text{Re} (\bar{H} \cdot \bar{H}^*)$$

but fast

$$U_{en} = \frac{\frac{\omega \epsilon \beta b}{4 h^2} A_u^2}{\frac{\epsilon b}{4 h^2} k^2 A_u^2} = \frac{\omega \beta}{k^2}$$

$$(W_e)_{ave} = \frac{\epsilon}{4} A_u^2 \left\{ \sin^2\left(\frac{n\pi y}{b}\right) + \frac{\beta^2}{h^2} \cos^2\left(\frac{n\pi y}{b}\right) \right\}$$

$$\rightarrow \int_0^b (W_e)_{ave} dy = \frac{\epsilon b}{8} A_u^2 \left\{ 1 + \frac{\beta^2}{h^2} \right\}$$

$$= \frac{\epsilon b}{8 h^2} A_u^2 \{ h^2 + \beta^2 \} = \frac{\epsilon b}{8 h^2} k^2 A_u^2$$

$$= \frac{\omega}{k} \cdot \frac{\beta}{k}$$

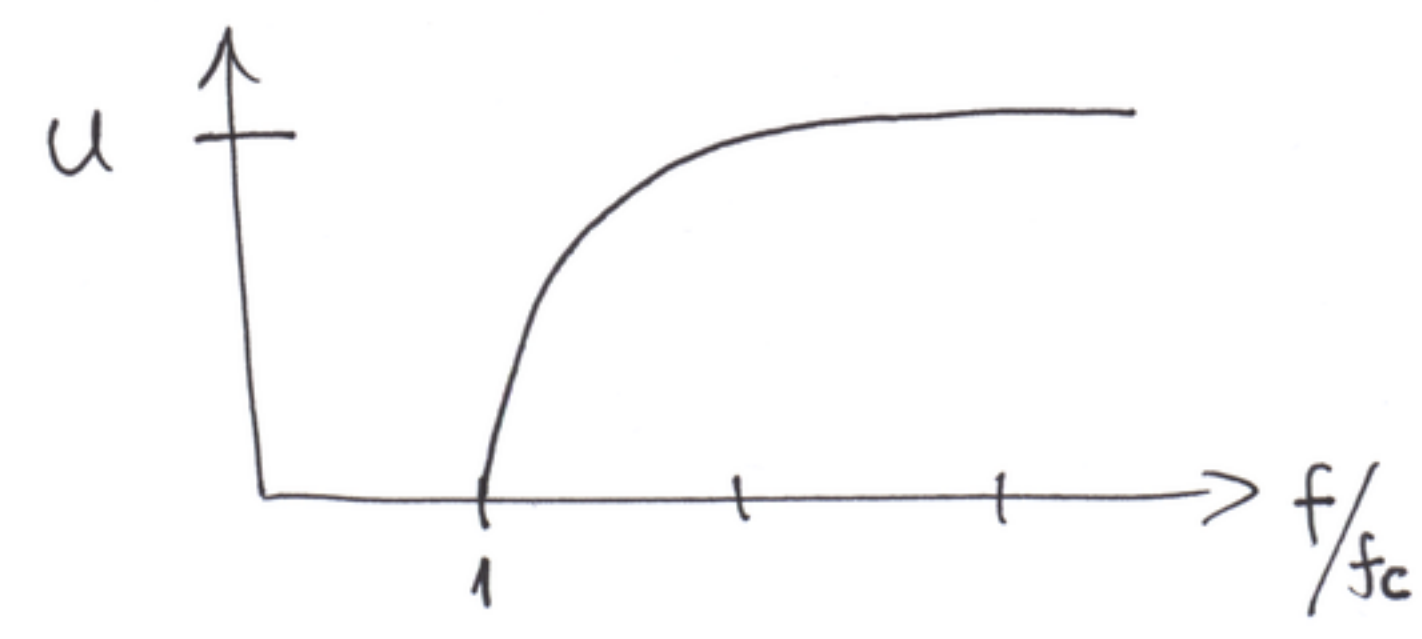
$$= \frac{\omega}{\omega \sqrt{\mu \epsilon}} \cdot \frac{k \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}{k}$$

$$= \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$= U_g$$

og sinus fyrir segulsuðid

$$\int_0^b (W_m)_{ave} dy = \frac{\epsilon b}{8 h^2} k^2 A_u^2$$



Rétthyruder bylgjuleiðari

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TM-bylgjur

$$H_z = 0$$

$$E_z(x, y, z) = E_z^0(x, y) e^{-\gamma z}$$

með

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2 \right) E_z^0(x, y) = 0$$

Lögunum leiðir aðgreiningu breytistofna

$$E_z^0(x, y) = X(x) Y(y)$$

pá fæst

$$X''(x) + k_x^2 X(x) = 0$$

$$Y''(y) + k_y^2 Y(y) = 0$$

með $k_y^2 + k_x^2 = h^2$

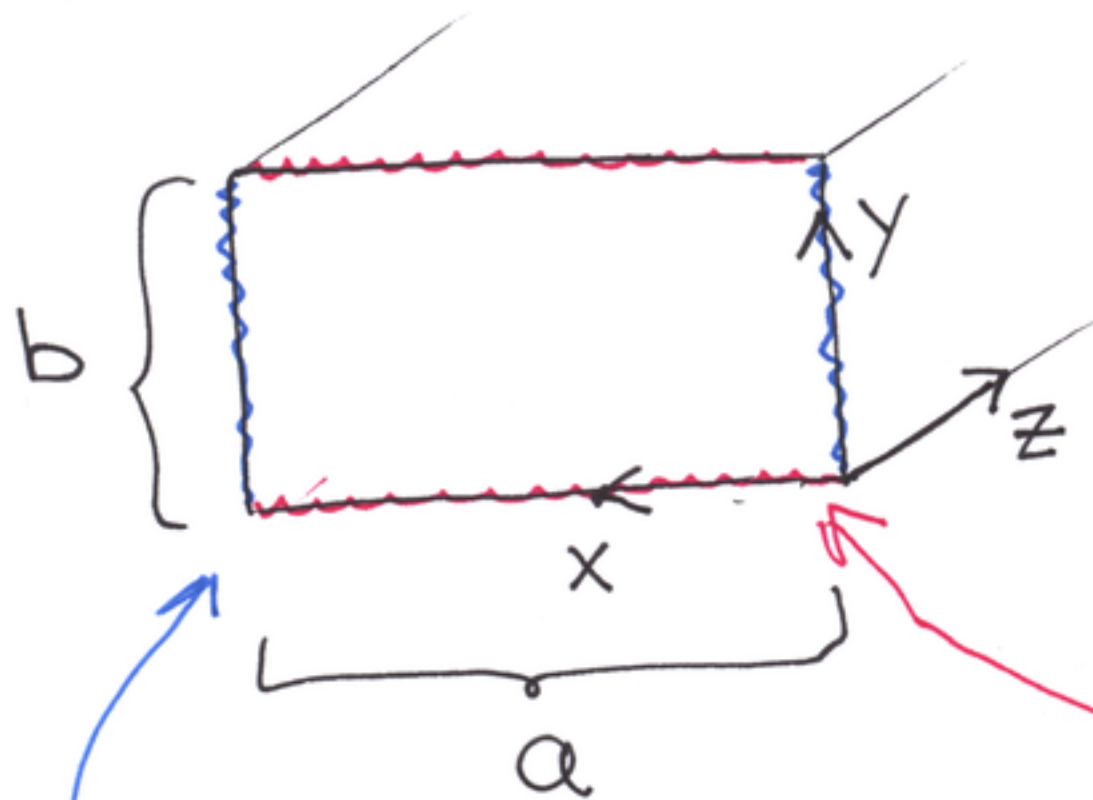
þaðar gildir

$$E_z^0(0, y) = 0$$

$$E_z^0(a, y) = 0$$

$$E_z^0(x, 0) = 0$$

$$E_z^0(x, b) = 0$$



Ennulausnirnar fyrir X og Y
verða $\sin k_x x, \sin k_y y$

með

$$k_x = \frac{m\pi}{a}, \quad m = 1, 2, 3, \dots$$

$$k_y = \frac{n\pi}{b}, \quad n = 1, 2, 3, \dots$$

$$\rightarrow E_z^0(x, y) = E_0 \sin(k_x x) \sin(k_y y)$$

og

$$k^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = i\beta = i \sqrt{k^2 - h^2} \\ = i \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

)/ TIL viðbótar fast ⑥

$$E_x^0(x, y) = -\frac{\gamma}{h^2} k_x E_0 \cos(k_x x) \sin(k_y y)$$

$$E_y^0(x, y) = -\frac{\gamma}{h^2} k_y E_0 \sin(k_x x) \cos(k_y y)$$

$$H_x^0(x, y) = \frac{i\omega \epsilon}{h^2} k_y E_0 \sin(k_x x) \cos(k_y y)$$

$$H_y^0(x, y) = -\frac{i\omega \epsilon}{h^2} k_x E_0 \cos(k_x x) \sin(k_y y)$$

$$(f_c)_{mn} = \frac{1}{2\mu\epsilon} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$(\lambda_c)_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

lesa sjálfum TE í
rétthyrndum leiðar

Þar kemur í ljós að
TE₁₀ er ríkjandi háttur
í rétthyrndum leiðum
ef $a > b$

Hringlaga bylgja stætur

Hútakerfið er sívalningskútt

$$\vec{E} = \vec{E}_T + \hat{a}_z E_z$$

$$\vec{H} = \vec{H}_T + \hat{a}_z H_z$$

↑ þverþættur

TEM-bylgjur eru ekki til
í þeim

→ TE og TM-bylgjur

$$\nabla_{r\phi}^2 E_z^0 + (\gamma^2 + k^2) E_z^0 = 0$$

$$E_z = E_z^0 e^{-\gamma z}$$

$$E_z(r, \phi, z) = E_z^0(r, \phi) e^{-\gamma z}$$

Lausnir

$$E_z^0(r, \phi) = C_n J_n(kr) \cos(n\phi)$$

Bessel fall

$$(E_T^0)_{TM} = \hat{a}_r E_r^0 + \hat{a}_\phi E_\phi^0 = -\frac{\gamma}{h^2} \nabla_T E_z^0$$

$$= -\frac{\gamma}{h^2} \left(\hat{a}_r \frac{\partial}{\partial r} + \hat{a}_\phi \frac{1}{r} \frac{\partial}{\partial \phi} \right) E_z^0$$

Eigingildi TM
hättanna finnst
frá

$$J_n(ha) = 0$$

$$\rightarrow E_r^0 = -\frac{i\beta}{h} C_n J_n'(hr) \cos(n\phi)$$

$$E_\phi^0 = \frac{i\beta n}{h^2 r} C_n J_n(hr) \sin(n\phi)$$

$$H_r^0 = -\frac{i\omega\epsilon n}{h^2 r} C_n J_n(hr) \sin(n\phi)$$

$$H_\phi^0 = -\frac{i\omega\epsilon}{h} C_n J_n'(hr) \cos(n\phi)$$

$$H_z^0 = 0$$

$$\text{því } E_z^0(a, \phi) = 0$$

Gildin á mögulegum
"h" ákvarðast frá
núlstöðum J_n

lægsta núlstaðin er fyrir

$J_0 \rightarrow$ TM₀₁-hattur er lagður hér

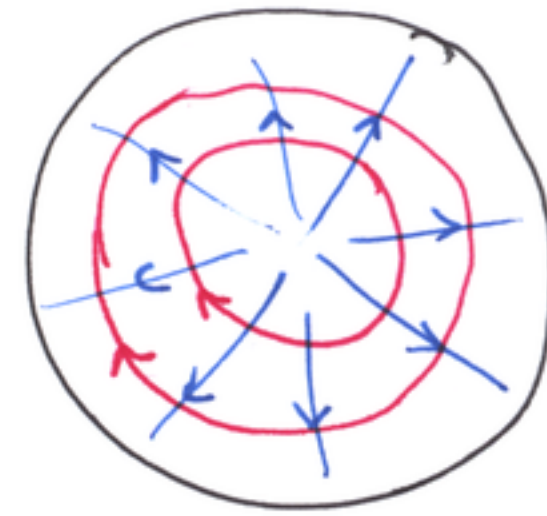
lesa sjálf um TE-bylgjur
í sívalungsbylgjuþotki

þar kemur í ljós að $J'_n(ka) = 0$

hefur lágstu rót fyrir $n=1$

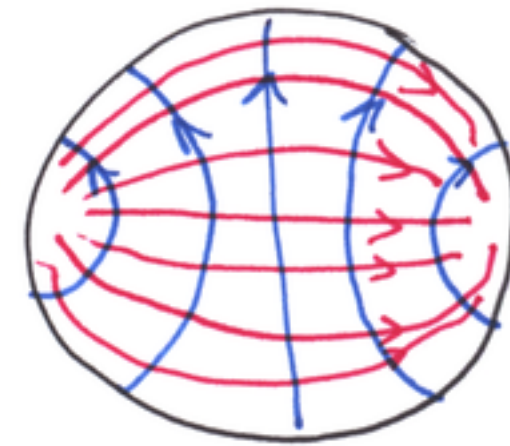
→ TE_{11} er lágsti hættur

og er útgandi hættur í
hringlaga leiðum



H
E

TM_{01}



TE_{11}

Bessel föll

1a)

$$u'' + \frac{1}{z} u' + \left(1 - \frac{\nu^2}{z^2}\right) u = 0$$

Lösningar är lika till för
rannigild z og jafnvel
tvinnigild.

hefur lösningar

$$u = A_n J_n(z) + B Y_n(z)$$

↑ serstödur. i $z=0$

fyrir $\nu=n=0, 1, 2, \dots$

$$J_n'(z) = \frac{1}{2} \{ J_{n-1}(z) - J_{n+1}(z) \} \quad n=1, 2, \dots \quad J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z} J_n(z)$$

$$J_0'(z) = -J_1(z)$$

Bessel Fall

