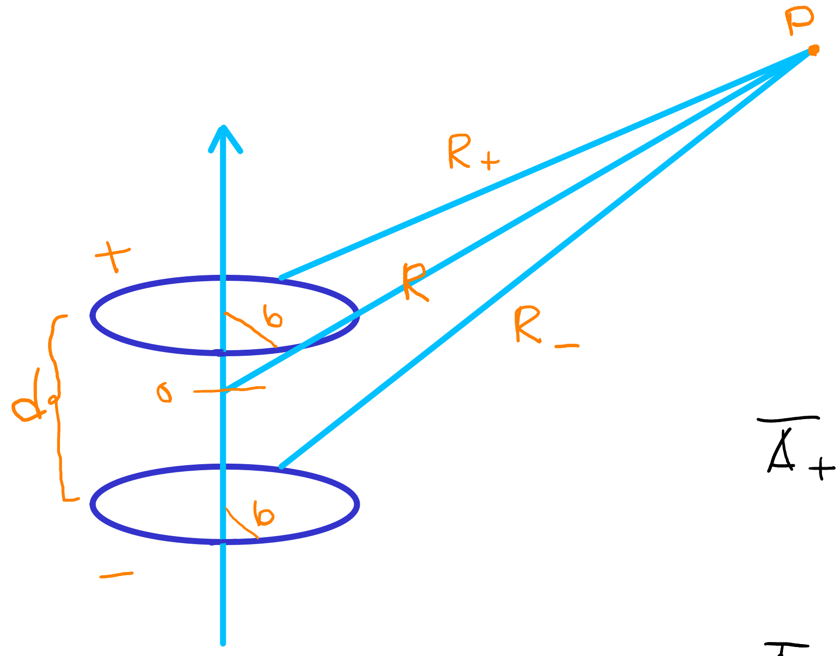
Problem 1



$b, d \ll R$

$$i_+(t) = +I_0 \cos(\omega t)$$

$$i_+(t) = -I_0 \cos(\omega t)$$

$$\bar{A}_+ = \hat{a}_\phi \frac{\mu_0 m}{4\pi R_+^2} (1 + i\beta R_+) e^{-i\beta R_+} \sin\theta_+$$

$$\bar{A}_- = -\hat{a}_\phi \frac{\mu_0 m}{4\pi R_-^2} (1 + i\beta R_-) e^{-i\beta R_-} \theta_-$$

We set $\theta_+ \sim \theta_-$, but

$$R_\pm = \sqrt{R^2 + \left(\frac{d}{2}\right)^2 \mp Rd\cos\theta} \approx \left(R \mp \frac{d}{2}\cos\theta\right)$$

$$\longrightarrow \quad \bar{A}_\pm \simeq \pm \hat{a}_\phi \frac{\mu_0 m}{4\pi R^2} (1 + i\beta R) \sin\theta \, e^{-i\beta R} e^{\pm \frac{i\beta d}{2}\cos\theta}$$

$$\rightarrow \quad \bar{A} = \hat{a}_\phi \frac{2i\mu_o w}{4\pi R^2}(1+i\beta R)\sin\theta\, e^{-i\beta R}\sin\left(\frac{\beta d}{2}\cos\theta\right)$$

thus

$$\bar{E} \simeq \hat{a}_\phi \frac{\omega\mu_o w}{2\pi R^2}(1+\underbrace{i\beta R}_{})\sin\theta\, e^{-i\beta R}\sin\left(\frac{\beta d}{2}\cos\theta\right)$$

leading to the far-field (E)

$$\bar{E} \simeq \hat{a}_\phi \frac{\omega\mu_o w}{2\pi R} i\beta\, e^{-i\beta R}\sin\theta\sin\left(\frac{\beta d}{2}\cos\theta\right)$$

$$\bar{H} = \frac{1}{\mu_o}\bar{\nabla}\times\bar{A}$$

$$\rightarrow \begin{cases} H_r = \dfrac{1}{\mu_o}\dfrac{1}{R\sin\theta}\dfrac{\partial}{\partial\theta}(A_\phi\sin\theta) \\[2ex] H_\theta = -\dfrac{1}{\mu_o R}\dfrac{\partial}{\partial R}(R A_\phi) \end{cases}$$

The radiation pattern is found from the angular part of the far-electric field

$$E = \hat{a}_\phi \frac{\omega \mu_0 m}{2\pi R} i\beta e^{-i\beta R} F(\theta)$$

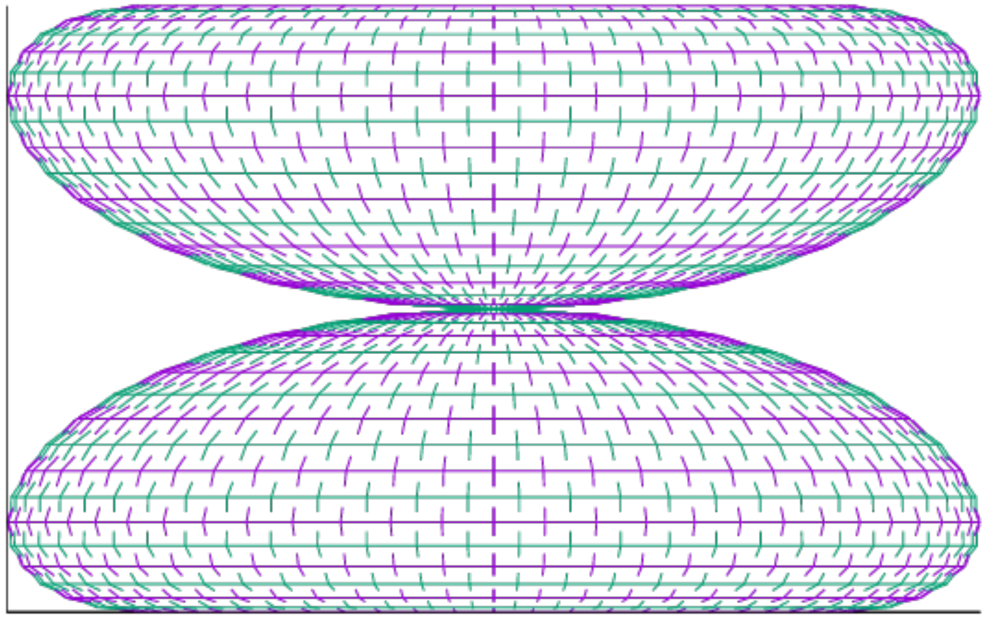
$$\rightarrow F(\theta) = \sin\theta \cdot \sin\left(\frac{\beta d}{2}\cos\theta\right), \quad \frac{\beta d}{2} = \frac{2\pi d}{2\lambda} \ll 1$$

The wavelength is considered much larger than b

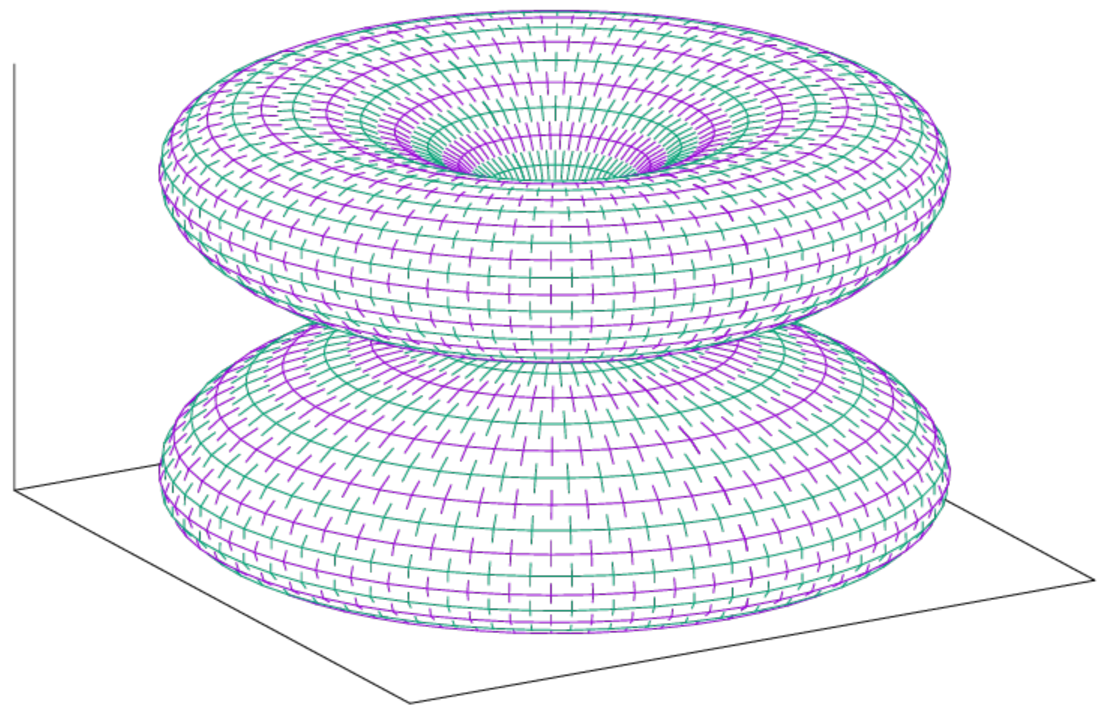
$$\rightarrow F(\theta) \simeq \sin\theta \cdot \frac{\beta d}{2}\cos\theta = \left(\frac{\beta d}{2}\right)\sin\theta\cos\theta$$

$$= \left(\frac{\beta d}{2}\right)\frac{1}{2}\sin(2\theta)$$

$F(\theta)$



$F(\theta)$



$F(\theta)$