$R_\ell(r)$ verður þá að uppfylla radial jöfnuna

$$\left\{ \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{\ell(\ell+1)}{r^2} + k^2 - U(r) \right\} R_\ell(r) = 0$$

Ef $rU(r) \xrightarrow[r\to 0]{} C \in \mathbb{R}$ þá er lausnin valin endanleg fyrir $r=0$ $\quad R_\ell \xrightarrow[r\to 0]{} A r^\ell$

$\rightarrow$ <u>nær lausnin</u> $R_\ell \sim j_\ell$

<u>fjarri markinu</u> $\quad r^2 U(r) \xrightarrow[r\to\infty]{} 0$

verður lausnin (<u>þar getur ekki V(r)</u>)

$$R_\ell(r) = B_\ell \left\{ h_\ell^{(2)}(kr) + S_\ell(k)\, h_\ell^{(1)}(kr) \right\}$$

$$\xrightarrow[r\to\infty]{} \quad \frac{1}{kr} e^{ikr\ldots} \qquad\qquad \frac{1}{kr} e^{-ikr}$$

kúlubylgja $\quad$ <u>inn</u> $\quad$ <u>út</u>

innbylgjuna mátti skrifa sem

$$e^{i\bar{k}\cdot\bar{r}} = \sum_{\ell=0}^{\infty} i^\ell (2\ell+1)\, j_\ell(kr)\, P_\ell(\cos\theta)$$

$$= \frac{1}{2}\sum_{\ell=0}^{\infty} i^\ell (2\ell+1) \left\{ h_\ell^{(1)}(kr) + h_\ell^{(2)}(kr) \right\} P_\ell(\cos\theta)$$

þessa vegna er heppilegt að velja heildar lausnina fjarri markinu (aðfellunni) með $B_\ell = \frac{1}{2}$:

$$\psi(\bar{r}) \xrightarrow[r\to\infty]{} \sum_{\ell=0}^{\infty} i^\ell (2\ell+1) \frac{1}{2} \left\{ h_\ell^{(2)}(kr) + S_\ell(k)\, \underset{\substack{\uparrow \\ \text{útbylgja}}}{h_\ell^{(1)}(kr)} \right\} P_\ell(\cos\theta)$$

því þá eru <u>öll</u> áhrif dreifingarinnar í $S_\ell(k)$

$$\psi(\bar{r}) \xrightarrow[r\to\infty]{} \frac{1}{2}\sum_{l=0}^{\infty} i^l(2l+1)\Big\{ h_l^{(1)}(kr) + h_l^{(2)}(kr)$$

$$+ \left(S_l(k)-1\right) h_l^{(1)}(kr)\Big\} P_l(\cos\theta)$$

$$= \sum_{l=0}^{\infty} i^l(2l+1)\left\{ j_l(kr) + \tfrac{1}{2}\left(S_l(k)-1\right) h_l^{(1)}(kr)\right\} P_l(\cos\theta)$$

$$= \underbrace{e^{i\bar{k}\cdot\bar{r}}}_{\text{inn bylgja}} + \sum_{l=0}^{\infty} i^l(2l+1)\tfrac{1}{2}\left(S_l(k)-1\right) h_l^{(1)}(kr) P_l(\cos\theta)$$

inn bylgja

plansbylgja

kúlubylgja út

nota aðfella formið

$$h_l^{(1)}(kr) \xrightarrow[r\to\infty]{} \frac{1}{kr} e^{ikr - \frac{i}{2}(l+1)\pi}$$

bera saman við

$$\psi(\bar{r}) \xrightarrow[r\to\infty]{} e^{i\bar{k}\cdot\bar{r}} + f(k,\theta)\frac{e^{ikr}}{r}$$

þá fæst:

$$f(k,\theta) = \frac{i}{2k}\sum_{l=0}^{\infty}(2l+1)\left(1-S_l(k)\right)P_l(\cos\theta)$$

<u>fjaðrandi dreifing.</u>

l-bylgjur blandast aftur

↓

líkindin verða að varðveitast fyrir hvert l

↓

radial-flæði fyrir inn og út l-þátt ~~varðveitist~~. er eins.

radial flæði $e^{i\bar{k}\cdot\bar{r}}$ er núll
(plansbylgja)

$\longrightarrow$ radialflæði $i^l(2l+1)j_l(kr)$
hverfur líka

$R_l(r)$: hafði bæði inn og út flæði með leiðandi flæði 0

Á fellu form $R_l(r)$ notað í

$$0 = \lim_{r\to\infty} r^2 \frac{\hbar}{2im}\left\{R_l^* \frac{\partial R_l}{\partial r} - R_l \frac{\partial R_l^*}{\partial r}\right\}$$

leiðir til

$$\boxed{|S_l(k)|^2 = 1}$$

$$B_l\left\{h_l^{(2)}(kr) + S_l(k)\, h_l^{(1)}(kr)\right\}$$

$$= B_l\, e^{i\delta_l}\left\{(j_l - in_l)e^{-i\delta_l} + e^{i\delta_l}(j_l + in_l)\right\}$$

Fasavik

Því er eðlilegt að skilgreina fasavikið $\delta_\ell(k)$:

venja

$$S_\ell(k) = e^{2i\delta_\ell(k)} \quad , \delta_\ell \in \mathbb{R}$$

Athugum markmiðið með $U(r)=0 \quad r \geq a$ (endanleg setning). Þá er almenn lausn radial jöfnunnar f. $r > a$

$$R_\ell(r) = A_\ell \left\{ \cos\delta_\ell \, j_\ell(kr) - \sin\delta_\ell \, n_\ell(kr) \right\}$$

$$\xrightarrow[r\to\infty]{} \frac{A_\ell}{kr}\left\{ \cos\delta_\ell \sin\left(kr - \frac{\ell\pi}{2}\right) + \sin\delta_\ell \cos\left(kr - \frac{\ell\pi}{2}\right)\right\}$$

$$\xrightarrow[r\to\infty]{} \frac{A_\ell}{kr} \sin\left(kr - \frac{\ell\pi}{2} + \delta_\ell\right)$$

innbylgjan var

$$j_\ell(kr) \xrightarrow[r\to\infty]{} \frac{1}{kr}\sin\left(kr - \frac{\ell\pi}{2}\right)$$

$\longrightarrow$ $\boxed{\text{mælt er fasavik}}$

samkvæmt skilgreiningu fæst: (fæst)

$$f(k,\theta) = \frac{i}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)\left(1 - S_\ell(k)\right)P_\ell(\cos\theta)$$

en $1 - e^{2i\delta_\ell} = -2i\, e^{i\delta_\ell}\sin\delta_\ell$

$$\rightarrow f(k,\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)\, e^{i\delta_\ell}\sin\delta_\ell \, P_\ell(\cos\theta)$$

og því

$$|f(k,\theta)|^2 = \frac{1}{k^2}\sum_{\ell,\ell'=0}^{\infty}(2\ell+1)(2\ell'+1)\sin\delta_\ell \sin\delta_{\ell'} \cos(\delta_\ell - \delta_{\ell'})\, P_\ell(\cos\theta)\, P_{\ell'}(\cos\theta)$$

2 ?

þó svo að δ_l finnist er tvöfalda summan oft erfið

$\longrightarrow$ athuga þart hve marga liði þart í raun

Í ljós mun koma að aðeins bylgjur með $l < ka$, a.: seinni mætti taka þátt í dreifingunni

---

Ljósfræðisetningin

finnum heildar þversnið

$$\sigma_t = \int d\Omega \frac{d\sigma}{d\Omega}$$

notum

$$\int d\Omega \, P_l(\cos\theta) P_{l'}(\cos\theta) = \frac{4\pi}{2l+1} \delta_{l,l'}$$

ögn með $l < ka$ sér ekki $V(r)$

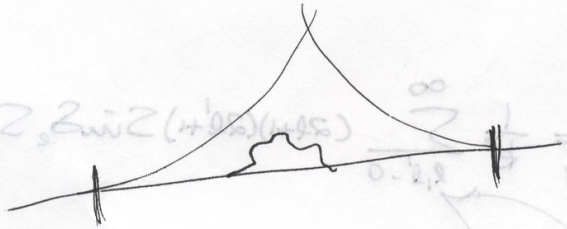
ef $\boxed{r \geq a \text{ má sleppa } V(r)}$

$$E_{tot} = E_{kin} = \frac{\hbar^2 k^2}{2m}$$

$$E_{kin} \geq E_{orbit} = \frac{\hbar^2 l(l+1)}{2ma^2}$$

$$\rightarrow k^2 a^2 \geq l(l+1) \quad \text{ef } r = a$$

$$\rightarrow \boxed{l < ka}$$



$$f(k,\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\, e^{i\delta_l}\sin\delta_l\, P_l(\cos\theta)$$

$$|f(k,\theta)|^2 = \frac{1}{k^2}\sum_{l,l'=0}^{\infty}(2l+1)(2l'+1)\sin\delta_l \sin\delta_{l'}$$

$$\cos(\delta_l-\delta_{l'})\, P_l(\cos\theta)\, P_{l'}(\cos\theta)$$

$$e^{i\delta_l}\sin\delta_l = \frac{1-e^{2i\delta_l}}{2i}$$

$$= \frac{1-S(k)}{2i}$$

$$\rightarrow \sigma_t = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l$$

eins má finna

$$\frac{d\sigma}{d\Omega} = |f(\ldots)|^2$$

$$\mathrm{Im}\, f(k,\theta) = \frac{1}{2k}\sum_{l=0}^{\infty}(2l+1)\,\mathrm{Re}\,(1-S_l)\,P_l(\cos\theta)$$

$$= \frac{1}{2k}\sum_{l=0}^{\infty}(2l+1)\,2\sin^2\delta_l\, P_l$$

Setjum $\theta=0$ og notum $P_l(1)=1$

$$\rightarrow \mathrm{Im}\, f(k,0) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l$$

samanburður gefur

$$\boxed{\sigma_t = \frac{4\pi}{k}\,\mathrm{Im}\, f(k,0)}$$

$\rightarrow$ $\mathrm{Im} f(k,0)$ segir til um hversu margar eindir dreifast úr innfallsstefnu

---

Heildarþversnið

$$\sigma_t = \sum_{l=0}^{\infty} \sigma_l^t$$

$$\rightarrow \quad \sigma_l^t = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l$$

fjaðrandi árekstrar $\rightarrow \delta_l \in \mathbb{R}$

$$\rightarrow \quad \sigma_l^t \leq \frac{4\pi}{k^2}(2l+1)$$

<u>efri mörk</u> nást aðeins þegar

$$\delta_l = (n + 1/2)\pi$$

<u>Dæmi : Orkubrunnur</u>

$$V(r) = \begin{cases} -V_0 & V_0 > 0 \quad r \leq a \\ 0 & r > a \end{cases}$$

$$K_0^2 = \frac{2m}{\hbar^2} V_0 \quad , \quad k^2 = \frac{2m}{\hbar^2} E$$

$$K^2 = k^2 + K_0^2$$

<u>Radíal jöfnurnar eru</u>

$$\left\{ \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} + K^2 \right\} R_l(r) = 0, \quad \underline{\underline{r \leq a}}$$

$$\left\{ \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} + k^2 \right\} R_l(r) = 0, \quad \underline{\underline{r > a}}$$

Lausnirnar með $R_\ell(0)$ endanlegt og að $R_\ell(r)$ samsvari föstum flæði fyrir $r \to \infty$ eru

$$R_\ell(r) = A_\ell j_\ell(Kr) \leftarrow \underline{\underline{r \leq a}}$$

$$R_\ell(r) = j_\ell(kr) + \tfrac{1}{2}(S_\ell(k)-1)\, h_\ell^{(1)}(kr) \quad \underline{\underline{r > a}}$$

nú er $h_\ell^{(1)} = j_\ell + i n_\ell$, $S_\ell(k) = e^{2i\delta_\ell(k)}$

$$\rightarrow R_\ell(r) = j_\ell + i e^{i\delta} \sin\delta\, h_\ell$$

$$= j_\ell + i e^{i\delta} \sin\delta \cdot (j_\ell + i n_\ell)$$

$$= j_\ell (i e^{i\delta} \sin\delta + 1) - e^{i\delta} \sin\delta \cdot n_\ell$$

$$= e^{i\delta} \left\{ j_\ell (e^{-i\delta} + i \sin\delta) - \sin\delta \cdot n_\ell \right\}$$

$$= e^{i\delta_\ell} \left\{ \cos\delta_\ell \cdot j_\ell(kr) - \sin\delta_\ell \cdot n_\ell(kr) \right\}$$

<u>Skeyta saman lausnum við $r=a$</u>

$$\frac{K j_\ell'(Ka)}{j_\ell(Ka)} = \frac{k\left\{ j_\ell'(ka)\cos\delta_\ell - n_\ell'(ka)\sin\delta_\ell \right\}}{j_\ell(ka)\cos\delta_\ell - n_\ell(ka)\sin\delta_\ell}$$

$$\rightarrow \tan\delta_\ell = \frac{k j_\ell(Ka)\, j_\ell'(ka) - K j_\ell'(Ka)\, j_\ell(ka)}{k j_\ell(Ka)\, n_\ell'(ka) - K j_\ell'(Ka)\, n_\ell(ka)}$$

athugum áreksturs við mjög lága orku

$\longrightarrow$ $\ell = 0$ s-bylgja er mikilvæg

$$j_0(kr) = \frac{\sin kr}{kr}$$

$$n_0(kr) = -\frac{\cos kr}{kr}$$

$$\rightarrow \tan\delta_0 = \frac{k\tan Ka - K\tan ka}{K + k\tan Ka \cdot \tan ka}$$

$$= \tan\left\{\arctan\left(\frac{k}{K}\tan ka\right) - ka\right\}$$

$$\rightarrow \delta_0 = \arctan\left\{\frac{k}{K}\tan ka\right\} - ka$$

Ef $V_0 \rightarrow 0$ $E \rightarrow 0$

$\rightarrow ka \ll 1$ $Ka \ll 1$

$$\rightarrow \delta_0 \simeq ka\left(\frac{\tan ka}{ka} - 1\right) \xrightarrow[k\to 0]{} 0$$

Ef $E \rightarrow 0$ en $V_0 \not\rightarrow 0$

$$\rightarrow \delta_0 \simeq ka\left(\frac{\tan ka}{ka} - 1\right)$$

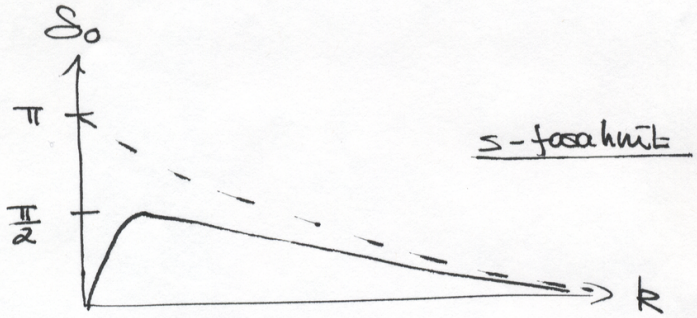
fyrir $ka < \frac{\pi}{2}$

þegar k vex bilar nálgunin er $Ka \rightarrow \frac{\pi}{2}$, í raun skiptir δ_0 um formerki

k er lítið svo þetta gerist aðeins ef $K_0 a \lesssim \frac{\pi}{2}$

ef $K_0 a > \pi/2$ þá mun $\delta_0 \rightarrow \pi$, $k \rightarrow 0$

$$\delta_0 = \pi + ka\left(\frac{\tan Ka}{Ka} - 1\right)$$



$K_0 a = \frac{\pi}{2}$: er minnsti styrkur mattis til þess að hafa eitt bundið ástand

Þetta hefur verið alhæft:

Levinsons setningin

Fyrir $E=0$ er s-fasavikið

$$\delta_0(0) = n\pi$$

þar sem n: er fjöldi s-bundinna ástanda

Ef bundna ástandið er aðeins stöðugt ástand þá gildir

$$\delta_0(0) = (n+\tfrac{1}{2})\pi$$

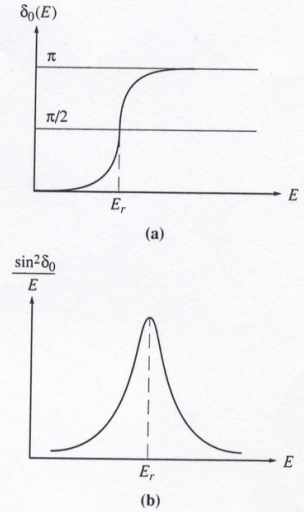


FIGURE 23.10
When the phase shift increases through $\pi/2$ (a), the corresponding partial wave cross section achieves a resonance (b).

The total cross section approaches a maximum value (fig. 23.10). This is called a resonance (see below). If there is a bound state at zero energy and the phase shift goes through $\pi/2$, the cross section peaks, and we have a resonance.

Breit-Wigner Formula

Let's examine the behavior of a cross section near a resonance. At the resonant energy E_r, the phase shift increases through $\pi/2$, and we have

$$\cos\delta_0(E_r) = 0 \qquad \sin\delta_0(E_r) = 1$$

At $E \sim E_r$, we can expand $\sin\delta_0$ and $\cos\delta_0$ by means of Taylor series:

$$\sin\delta_0(E) = \sin\delta_0(E_r) + \cos\delta_0(E)\left.\frac{d\delta_0(E)}{dE}\right|_{E=E_r}(E-E_r)$$

$$= 1 \qquad (23.54)$$

$$\cos\delta_0(E) = \cos\delta_0(E_r) - \sin\delta_0(E)\left.\frac{d\delta_0(E)}{dE}\right|_{E=E_r}(E-E_r)$$
$$= -\left.\frac{d\delta_0(E)}{dE}\right|_{E=E_r}(E-E_r) = -\frac{2}{\Gamma}(E-E_r) \qquad (23.55)$$

The last equation defines Γ, which we will interpret a little later.

The scattering amplitude is given by

$$f_0(\theta,E) = (1/k)\exp[i\delta_0(E)]\sin\delta_0(E) \qquad (23.56)$$

The $1/k$-variation with energy is slow and uninteresting. The interesting, rapidly varying part comes from the phase shifts; let's call this part $f_0(\delta)$. We have

$$f_0(\delta) = e^{i\delta_0(E)}\sin\delta_0(E) = \frac{\sin\delta_0(E)}{\cos\delta_0(E) - i\sin\delta_0(E)}$$
$$\approx \frac{1}{-2(E-E_r)/\Gamma - i} = -\frac{\Gamma/2}{(E-E_r)+i\Gamma/2} \qquad (23.57)$$

where we have used the Taylor-expanded values of $\cos\delta_0$ and $\sin\delta_0$. The total cross section is now given by

$$\sigma_T = \frac{4\pi}{k^2}|f_0(\delta)|^2 = \frac{4\pi}{k^2}\frac{\Gamma^2/4}{(E-E_r)^2+\Gamma^2/4} \qquad (23.58)$$

which is the Breit-Wigner resonance formula (the resonance curve is shown in fig. 23.10b). Clearly, Γ represents the width of the resonant curve.

Low-Energy Neutron-Proton Scattering: Scattering Length and Effective Range

Suppose we consider the scattering from a square-well potential with enough depth that there is a bound state with a small binding energy. This is a model for neutron-proton scattering; the neutron-proton system does indeed have a bound state near zero energy, namely deuteron. The square-well treatment of deuteron was given in chapter 12, where we derived the following equation (changing the notation slightly):

$$k'_{\text{in}}\cot k'_{\text{in}}a = -k_B \qquad (12.37)$$

with

$$k'^2_{\text{in}} = 2\mu(V_0 - |E|)/\hbar^2 \qquad k_B^2 = 2\mu|E|/\hbar^2$$

tímaháð dreifing

aðfelluástönd

$$H = \frac{p^2}{2m} + V(r)$$

$V(r)$: skammsæilið

$$\longrightarrow \quad H \xrightarrow[r\to\infty]{} H_0 = \frac{p^2}{2m}$$

gerum ráð fyrir að til séu aðfelluástönd

norm í Hilbertrúmi sterk stöðugrði

$$\lim_{t\to-\infty} \| |\psi(t)\rangle - |\psi_{in}(t)\rangle \| = 0$$

$$\lim_{t\to\infty} \| |\psi(t)\rangle - |\psi_{out}(t)\rangle \| = 0$$

H og H_0 eru sjálfoka

-> til eru einoka prómar virkjar

$$U(t) = e^{-iHt/\hbar}$$

$$U_0(t) = e^{-iH_0t/\hbar}$$

Af felle skilyrði verður þá

$$\lim_{t \to \mp\infty} \| e^{-iHt/\hbar} |\psi(0)\rangle - e^{-iH_0t/\hbar} |\psi_{\substack{in\\out}}(0)\rangle \| = 0$$

Eða (einoka)

$$\lim_{t \to \mp\infty} \| e^{+iH_0t/\hbar} e^{-iHt/\hbar} |\psi(0)\rangle - |\psi_{\substack{in\\out}}(0)\rangle \| = 0 \qquad (*)$$

þurf þart að kanna eiginleika Møller-bylgjuvirkjanna

$$\Omega^{+}_{\pm} = \lim_{t \to \mp\infty} \left\{ e^{iH_0t/\hbar} e^{-iHt/\hbar} \right\}$$

---

samkvæmt (*) gildir

$$|\psi_{in}(t)\rangle = \Omega^{+}_{+} |\psi(t)\rangle$$

$$|\psi_{out}(t)\rangle = \Omega^{+}_{-} |\psi(t)\rangle$$

og einnig

$$|\psi(t)\rangle = \Omega_{+} |\psi_{in}(t)\rangle$$

$$= \Omega_{-} |\psi_{out}(t)\rangle$$

Ef tekið er saman fæst

$$|\Psi_{in}(t)\rangle = \Omega_+^+ \Omega_+ |\Psi_{in}(t)\rangle$$

$$|\Psi_{out}(t)\rangle = \Omega_-^+ \Omega_- |\Psi_{out}(t)\rangle$$

Gerum ráð fyrir að inn og út ástönd spanni allt Hilbertrúm dreifiástanda.

$$\longrightarrow \Omega_\pm^+ \Omega_\pm = 1$$ í dreifirúmum

þó er ekki hægt að álíta að $\Omega_\pm$ séu einaka, því ef H hefur bundin ástönd þá spanna inn og út ástöndin ekki allt Hilbertrúmið.

Athugum Schrödingerjöfnuna

$$i\hbar \partial_t |\Psi(t)\rangle = H|\Psi(t)\rangle$$

$$i\hbar \partial_t |\Psi_{\substack{in\\out}}(t)\rangle = H_0 |\Psi_{\substack{in\\out}}(t)\rangle \qquad (*)$$

$$i\hbar \partial_t \Omega_\pm |\Psi_{\substack{in\\out}}(t)\rangle = H\, \Omega_\pm |\Psi_{\substack{in\\out}}(t)\rangle$$

bera saman við $(*)$

$$\longrightarrow \boxed{H\Omega_\pm = \Omega_\pm H_0}$$

einsma frá $|\Psi_{out}(t)\rangle = \Omega_-^+ |\Psi(t)\rangle$ og $|\Psi(t)\rangle = \Omega_+ |\Psi_{in}(t)\rangle$ fá

$$\boxed{\begin{gathered} |\Psi_{out}(t)\rangle = \Omega_-^+ \Omega_+ |\Psi_{in}(t)\rangle \\ S \equiv \Omega_-^+ \Omega_+ \end{gathered}}$$

S-virkinn tengir áðfellu ástönd fyrir $t = -\infty$ og $t = \infty$

dreifingin eða $\frac{d\sigma}{d\Omega}$ mun tengjast skörum út og inn ástandanna

þú verða allar upplýsingar um dreifinguna í S-virkjanum.

Framsetning S-virkjans <u>í grunni frjálsra ástanda</u> er S-fylkið.

Eins hefði mátt fá

$$|\Psi_{in}(t)\rangle = \Omega_+^\dagger \Omega_- |\Psi_{out}(t)\rangle$$
$$= S^\dagger |\Psi_{out}(t)\rangle$$

þar sem $\{|\Psi_{in}\rangle\}$ og $\{|\Psi_{out}\rangle\}$ eru fullkomnir grunnar

$\rightarrow$ S er einóta

$$S^\dagger S = S S^\dagger = 1$$

$\Omega_\pm^\dagger \Omega_\pm = 1$ fyrir dreifiástöndin

$H\Omega_\pm = \Omega_\pm H_0$ og $S = \Omega_-^\dagger \Omega_+$

$$\rightarrow H_0 \Omega_\pm^\dagger = \Omega_\pm^\dagger H_0$$

$$\Omega_-^\dagger H \Omega_+ = \Omega_-^\dagger \Omega_+ H_0 = S H_0$$

og $$H_0 \Omega_-^\dagger \Omega_+ = H_0 S = \Omega_-^\dagger H \Omega_+$$

$$\rightarrow \boxed{S H_0 = H_0 S}$$

$\rightarrow$ <u>fylkisstök S</u> með eigin ástöndum H_0 (frjálsum) <u>eru tímaóháð</u>

Greenföll

$$H = H_0 + V(r)$$

$$H_0 = \frac{p^2}{2m}$$

Skilgreinum Greenföll (tímahöð-tímaþróun)

$$(i\hbar\partial_t - H_0)\, G_0^{\pm}(t) = \delta(t)$$

$$(i\hbar\partial_t - H)\, G^{\pm}(t) = \delta(t)$$

með jaðarskilyrðum

<u>Seinkuð:</u> $G_0^{+}(t) = G^{+}(t) = 0 \quad t < 0$

<u>Flýtta:</u> $G_0^{-}(t) = G^{-}(t) = 0 \quad t > 0$

Lausnir eru

$$G_0^{+}(t) = -\frac{i}{\hbar} e^{-iH_0 t/\hbar}\, \theta(t)$$

$$G_0^{-}(t) = \frac{i}{\hbar} e^{-iH_0 t/\hbar}\, \theta(-t)$$

$$G^{+}(t) = -\frac{i}{\hbar} e^{-iHt/\hbar}\, \theta(t)$$

$$G^{-}(t) = \frac{i}{\hbar} e^{-iHt/\hbar}\, \theta(t)$$

$$\theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \qquad \theta(0^+) = 1, \quad \theta(0^-) = 0$$

Lausnirnar tengjast Møller virkjunum og kallast: (propagators)

því ef

$$(i\hbar\partial_t - H_0)\, |\Psi_0(t)\rangle = 0$$

þá fæst fyrir $\underline{t > t'}$

$$|\Psi_0(t)\rangle = i\hbar G_0^+(t-t')|\Psi_0(t')\rangle$$

eða ef $\underline{t < t'}$

$$|\Psi_0(t)\rangle = -i\hbar G_0^-(t-t')|\Psi_0(t')\rangle$$

þú eru Møller virkjarnir markgildi þessara :

$|\Psi(t')\rangle$ er lausn Schrödingerjöfnunar með matti $V(r) \neq 0$

skilgreinum (*)

$$|\Psi_0(t)\rangle = i\hbar G_0^+(t-t')|\Psi(t')\rangle$$

fyrir $t > t'$ þá er $|\Psi_0(t)\rangle$ lausn Schrödinger jöfnunar fyrir H_0

fyrir $t = t'$

$$|\Psi_0(t')\rangle = |\Psi(t')\rangle$$

þú

$$\lim_{t \to 0^+} G_0^+(t) = -\frac{i}{\hbar}$$

athugum hvað gerist þegar $t' \to -\infty$

Vitum

$$|\Psi(t')\rangle = -i\hbar G^-(t')|\Psi(0)\rangle \quad t' < 0$$

notum í (*)

$$|\Psi_0(t)\rangle = (i\hbar)(-i\hbar) G_0^+(t-t') G^-(t')|\Psi(0)\rangle$$

$$= e^{-iH_0(t-t')/\hbar}\theta(t-t') e^{-iHt'/\hbar}\theta(-t')|\Psi(0)\rangle$$

og þar sem

$$|\Psi_0(t)\rangle = e^{-iH_0 t/\hbar}|\Psi_0(0)\rangle$$

$\rightarrow$

$$|\Psi_0(0)\rangle = \theta(t-t')\theta(-t')\, e^{iH_0 t'/\hbar}\, e^{-iHt'/\hbar}\, |\Psi(0)\rangle$$

$t' \rightarrow -\infty$

$$\rightarrow \quad |\Psi_0(0)\rangle = \Omega_+^{+}\, |\Psi(0)\rangle$$

$$\rightarrow \quad |\Psi_0(t)\rangle = |\Psi_{in}(t)\rangle$$

og áður

lausn á Sch H

$$|\Psi_0(t)\rangle = i\hbar\, G_0^{+}(t-t')\, |\Psi(t')\rangle$$

<u>Heildisjafna</u>

Hægt er að finna heildisjöfnu fyrir $G^{\pm}(t)$ sem felli af $G_0^{\pm}(t)$

notum t.d.

$$(i\hbar\partial_t - H_0)\, G^{+}(t) = \delta(t) + V G^{+}(t)$$

$$(i\hbar\partial_t - H_0)\, G_0^{+}(t) = \delta(t)$$

drögum þá síðari frá þeirri fyrri

$$\rightarrow \quad i\hbar\partial_t \left\{ G^{+}(t) - G_0(t) \right\} = H_0 \left\{ G^{+}(t) - G_0^{+}(t) \right\} + V G^{+}(t)$$

með innsetningu má sannreyna að lausnin er

$$\left\{ G^{+}(t) - G_0^{+}(t) \right\} = \int_{-\infty}^{\infty} G_0^{+}(t-t')\, V G^{+}(t')\, dt'$$

$$\rightarrow \quad \boxed{G^{\pm}(t) = G_0^{\pm}(t) + \int_{-\infty}^{\infty} G_0^{+}(t-t')\, V G^{+}(t')\, dt'}$$

jafna á <u>Dyson</u> formi

$$\boxed{\text{Fourier umbreytingin m.t.t. } t \text{ gefur Lippmann-Schwinger jöfnuna}}$$

þegar G verkar á $|\Psi\rangle$

$$(i\hbar\partial_t - H_0)|\Psi\rangle = V|\Psi\rangle$$

látum heildisjöfnuna verka á $|\Psi\rangle$

$$\rightarrow \boxed{|\Psi^{\pm}(t)\rangle = |\Psi_{\substack{in\\out}}(t)\rangle + \int_{-\infty}^{\infty} G_0^{\pm}(t-t')V|\Psi^{\pm}(t')\rangle dt'}$$

$$\lim_{t\to\mp\infty} |\Psi^{\pm}(t)\rangle = |\Psi_{\substack{in\\out}}(t)\rangle$$

er innifalið í jöfnunni

$\rightarrow$ Källén-Yang-Feldman jafna

S-fylkið og dreifivísirinn

$$|\Psi_{in}\rangle = S^{+}|\Psi_{out}\rangle$$

$$|\Psi_{out}\rangle = S|\Psi_{in}\rangle$$

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frjálst ástand

stammtatölur

$$|\Psi_{in}(t)\rangle = |\Psi_0(\alpha,t)\rangle$$

$$\downarrow$$

$$|\Psi^{+}(\alpha,t)\rangle$$

$$\downarrow$$

$$|\Psi_{out}(t)\rangle$$ merkt með β

$\downarrow$ r

ástöndin hafa blandast

$\rightarrow \beta \neq \alpha$

α merkir virkja sem víxlast við H_0 en ekki H

Í $|\Psi^{+}(\alpha,t)\rangle$ merkir α einungis hvernig ástandið varð til úr $|\Psi_{in}\rangle$

Við viljum finna (í tíma) líkindi þess að $|\Psi_0(\beta,t)\rangle$ þróist úr $|\Psi^+(\alpha,t)\rangle$ eftir langann tíma

$$\lim_{t\to\infty} \langle \Psi_0(\beta,t)|\Psi^{(+)}(\alpha,t)\rangle$$

$$= \langle \Psi_0(\beta,t)|\Psi_{out}(t)\rangle$$

$$= \langle \Psi_0(\beta,t)|S|\Psi_{in}(t)\rangle$$

$$= \langle \Psi_0(\beta,t)|S|\Psi_0(\alpha,t)\rangle$$

á sama hátt hefði mátt spyrja hvernig $|\Psi_{out}(t)\rangle$ tengist $|\Psi^-(\beta,t)\rangle$ og þ. tengist

$$\langle \Psi_0(\beta,t)|S|\Psi_0(\alpha,t)\rangle = \langle \Psi^-(\beta,t)|\Psi^+(\alpha,t)\rangle$$

Lippmann Schwinger

$$\longrightarrow \quad |\Psi^{(\pm)}\rangle = |\Psi^{(0)}\rangle + (E-H\pm i\epsilon)^{-1}V|\Psi^{(0)}\rangle$$

$\beta \to E_f$

$\alpha \to E_i$

$$\underline{(E-H)|\Psi^\pm\rangle} = (E-H_0)|\Psi^0\rangle$$
$$= (E-H+V)|\Psi^0\rangle = \underline{(E-H)|\Psi^0\rangle + V|\Psi^0\rangle}$$

$$\to S_{fi} = \langle \Psi^-(E_f)|\Psi^+(E_i)\rangle$$

$$= \langle \Psi^+(E_f)|\Psi^+(E_i)\rangle$$

$$+ \left\{\langle \Psi^-(E_f)| - \langle \Psi^+(E_f)|\right\}|\Psi^+(E_i)\rangle$$

$$\to |\Psi^-(E_f)\rangle - |\Psi^+(E_f)\rangle = \left\{\frac{1}{E_f-H-i\epsilon}V - \frac{1}{E_f-H+i\epsilon}V\right\}|\Psi^0(E_f)\rangle$$

gerum ráð fyrir

$$\langle \Psi^\pm(E_f)|\Psi^\pm(E_i)\rangle = \delta_{fi} = \delta(E_f-E_i)\delta_{\alpha\beta}$$

↑ hornréttur einingar grunn

$\rightarrow$

$$S_{fi} = \delta_{fi} + \langle \Psi^0(E_f) | V \frac{1}{E_f - H + i\epsilon} - V \frac{1}{E_f - H - i\epsilon} | \Psi^+(E_i) \rangle$$

$$= \delta_{fi} + \left\{ \frac{\mathcal{P}}{E_f - E_i} - i\pi\delta(E_f - E_i) - \frac{\mathcal{P}}{E_f - E_i} \right.$$

$$\left. - i\pi\delta(E_f - E_i) \right\} \langle \Psi^0(E_f) | V | \Psi^+(E_i) \rangle$$

$$\rightarrow S_{fi} = \delta_{fi} - 2\pi i \delta(E_f - E_i) \langle \Psi^0(E_f) | V | \Psi^+(E_i) \rangle$$

ada

$$\boxed{\begin{aligned} S_{fi} &= \delta_{fi} - 2\pi i \delta(E_f - E_i) T_{fi} \\ T_{fi} &= \langle \Psi^{(0)}(E_f) | V | \Psi^+(E_i) \rangle \\ &\quad \uparrow \\ &\quad E_f = E_i \end{aligned}}$$

$$\rightarrow f(k\theta) = -\frac{1}{4\pi} T_{fi}$$